LAWS GOVERNING THE REGULAR HEATING MODE IN SOLIDS DURING TRANSIENT RADIATIVE INTERACTION

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An experimental study has been made concerning the regular heating mode of bodies during radiative interaction. A method of calculating the temperature fields in interacting bodies has been developed in the basis of relations derived here for the regular heating mode.

The subject under consideration has been dealt with earlier in [1-3]. For a further study of these phenomena, the authors have performed experiments concerned with the thermal regularization in radiatively interacting bodies. A schematic diagram of the test apparatus is shown in Fig. 1. The volume of the test chamber 2 0.17 m^3. Vacuum (down to 10^{-4}-10^{-5} mm Hg) was produced by a VN-1MG intermediate-vacuum pump and an N-2T high-vacuum pump. The thermocouple leads were brought out through a wall in the vacuum apparatus through a special gasket 8 with hermetic sealing. The test specimens were steel cylinders 3 and 5 thermally insulated along their lateral surfaces. Shields 6 were made of aluminum foil 0.1 mm thick. There were altogether eight such shields. Axial play of the shields was eliminated by round jaws pressing on the specimen from both ends. The back surfaces of cylinders 5 were insulated by an envelope of shields 4. Specimen 3 was heated with heaters 1, in the upper part of the chamber. After the prescribed temperature had been reached, a voltage was applied across hanger 7 causing it to overheat. Specimen 3 was then gradually (with the aid of a counterweight) dropped into the active position. The temperature field in the specimens was recorded by means of twelve thermocouples through a model ÉPP-9 potentiometer.

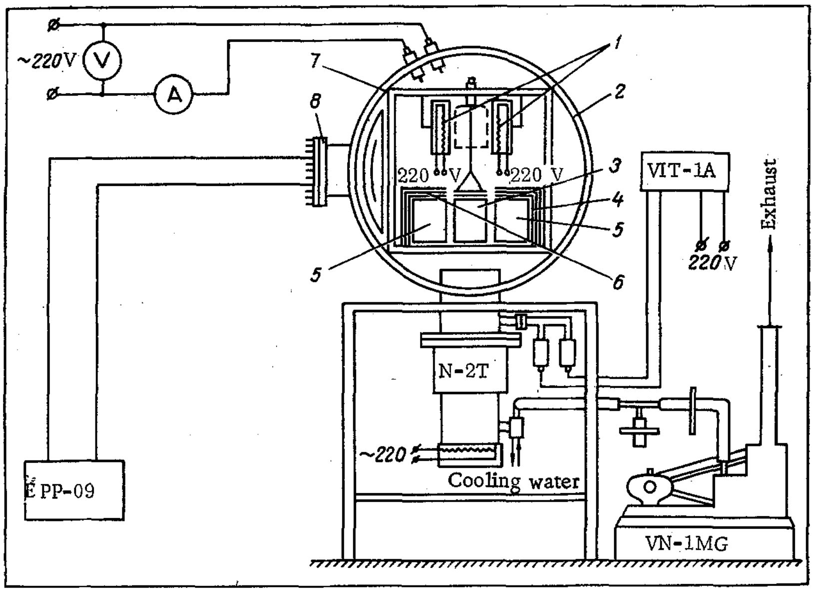

Fig. 1. Schematic diagram of a test stand for studying the transient radiative interaction between bodies.

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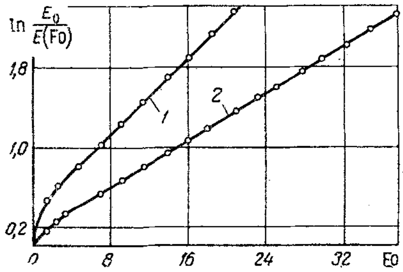

Fig. 2. Logarithm of the resultant flux as a function of time: 1) $\theta_{01} = 1.62$, $\theta_{02} = 0.762$, $k_a = 0.57$, $k_l = 0.4021$, $k_\lambda = 1.656$, $Sk_2 = 0.0133$; 2) $\theta_{01} = 1.67$, $\theta_{02} = 0.731$, $k_a = 1.00$, $k_l = 0.4021$, $k_\lambda = 1.000$, $Sk_2 = 0.0139$.

The test results with two interaction modes have been plotted in Fig. 2 in semilogarithmic coordinates. According to the graph, the regular mode occurs a certain time after heating has started. A distinguishing characteristic of this mode is that the natural logarithm of the resultant flux $\ln E(\mathrm{Fo})$ is a linear function of time. The rate of change of this logarithm remains constant throughout the transient period of radiative interaction.

With W denoting the rate at which the logarithm of the resultant flux changes, one can write

$$W = -\frac{\partial \ln E(\mathrm{Fo})}{\partial \mathrm{Fo}} = -\frac{1}{E(\mathrm{Fo})} \cdot \frac{\partial E(\mathrm{Fo})}{\partial \mathrm{Fo}} = \mathrm{const.} \tag{1}$$

We are looking for an approximate characterization of this mode and, for this purpose, we formulate the problem mathematically as follows:

$$\frac{\partial^2 \theta_1}{\partial X^2} = k_a \frac{\partial \theta_1}{\partial \mathrm{Fo}}; \quad \frac{\partial^2 \theta_2}{\partial X^2} = \frac{\partial \theta_2}{\partial \mathrm{Fo}}; \tag{2}$$

$$-k_l \leqslant X \leqslant 0; \quad 0 \leqslant X \leqslant 1;$$

$$\frac{\partial \theta_1(-k_l, \mathrm{Fo})}{\partial X} = 0; \quad \frac{\partial \theta_2(1, \mathrm{Fo})}{\partial X} = 0; \tag{3}$$

$$\frac{\partial \theta_1(0, \mathrm{Fo})}{\partial X} = -\mathrm{Sk}_1 \left[\theta_1^4(0, \mathrm{Fo}) - \theta_2^4(0, \mathrm{Fo})\right] = -E(\mathrm{Fo}); \tag{4}$$

$$\frac{\partial \theta_2(0, \mathrm{Fo})}{\partial X} = -\mathrm{Sk}_2 \left[\theta_1^4(0, \mathrm{Fo}) - \theta_2^4(0, \mathrm{Fo})\right] = -E(\mathrm{Fo}); \tag{5}$$

$$\theta_1(0, \mathrm{Fo}) = \theta_{01}; \quad \theta_2(0, \mathrm{Fo}) = \theta_{02}. \tag{6}$$

The boundary conditions (4) and (5) can also be expressed as

$$\theta_1(0, \mathrm{Fo}) = \theta_{1n}(\mathrm{Fo}), \quad \theta_2(0, \mathrm{Fo}) = \theta_{2n}(\mathrm{Fo}). \tag{7}$$

System (2)-(6) and system (2), (3), (6), (7) both describe the same process. Their simultaneous solution yields the following expression:

$$\ln E(\mathrm{Fo}) = -\frac{\pi^2}{4 k_a k_l^2}\mathrm{Fo} + \ln \frac{\pi^2}{2 k_a k_l^3}$$

$$+ \ln \left[\int_0^{\mathrm{Fo}} \theta_{1n}(\mathrm{Fo}) \exp \frac{\pi^2 \mathrm{Fo}}{4 k_a k_l^2} \cdot d\,\mathrm{Fo} - \frac{4\theta_{01} k_a k_l^2}{\pi^2} \exp\left[\frac{\pi^2 \mathrm{Fo}}{4 k_a k_l^2} + \frac{4\theta_{01} k_a k_l^2}{\pi^2}\right]\right]. \tag{8}$$

Within practical engineering accuracy, Eq. (8) can be approximated by a simpler equation

$$\ln \frac{E_0(\mathrm{Fo})}{E(\mathrm{Fo})} = \frac{\pi^2 \mathrm{Fo}}{4\left(1 + k_a k_l^2 \exp(-\mathrm{Sk}_\delta)\right)} + C. \tag{9}$$

Here Sk_δ is the larger of the two numbers Sk_1 or Sk_2. It is evident from Eqs. (10) and (9) that the $\pi^2/4(1 + k_a k_l^2 \exp(-\mathrm{Sk}_\delta))$ is the rate W at which the logarithm of the resultant flux changes. Equation (9) gives a rather adequate description of the regular stage of the process during a transient radiative interaction between bodies of finite dimensions. The applicability criterion for expression (9) is the quality of W calculated by the theoretical formula and found by tests. Both sets of values are compared in Table 1. Expression (9) is valid for bodies with $0.1 \leq \mathrm{Sk}_1 \leq 8$. The data for bodies with a smaller mass fit closely

TABLE 1. Comparison of Calculated Values for W

Problem conditions			W numerical analysis	W according to the formula	Δ, %
$\theta_{01}=1{,}4$ $k_a=1$	$\theta_{02}=0{,}8$ $k_l=2$	$Sk_1=5$ $Sk_2=0{,}125$ $k_\lambda=0{,}5$	0,721	0,720	0,139
$\theta_{01}=2{,}5$ $k_a=0{,}5$	$\theta_{02}=0{,}7$ $k_l=2$	$Sk_1=0{,}5$ $Sk_2=0{,}2$ $k_\lambda=0{,}2$	1,100	1,114	1,23
$\theta_{01}=2$ $k_a=1$	$\theta_{02}=0{,}5$ $k_l=1$	$Sk_1=1$ $Sk_2=0{,}5$ $k_\lambda=0{,}5$	1,81	1,803	0,387
$\theta_{01}=1{,}5$ $k_a=0{,}5$	$\theta_{02}=0{,}5$ $k_l=10$	$Sk_1=3$ $Sk_2=0{,}6$ $k_\lambda=0{,}2$	0,665	0,706	4,4
$\theta_{01}=1{,}5$ $k_a=1$	$\theta_{02}=0{,}6$ $k_l=1$	$Sk_1=0{,}4$ $Sk_2=0{,}32$ $k_\lambda=0{,}8$	1,471	1,477	0,407
$\theta_{01}=1{,}75$ $k_a=0{,}8$	$\theta_{02}=0{,}4$ $k_l=0{,}92$	$Sk_1=0{,}2$ $Sk_2=0{,}21$ $k_\lambda=1{,}0869$	1,551	1,590	2,45
$\theta_{01}=2$ $k_a=0{,}2$	$\theta_{02}=0{,}1$ $k_l=2{,}5$	$Sk_1=2$ $Sk_2=3{,}6$ $k_\lambda=1{,}8$	2,285	2,384	4,16
$\theta_{01}=2$ $k_a=0{,}46$	$\theta_{02}=0{,}1$ $k_l=1{,}1$	$Sk_1=0{,}5$ $Sk_2=0{,}889$ $k_\lambda=1{,}778$	2,09	2,008	3,9
$\theta_{01}=1{,}6$ $k_a=0{,}15$	$\theta_{02}=0{,}1$ $k_l=2$	$Sk_1=1$ $Sk_2=5$ $k_\lambda=5$	2,46	2,457	0,121
$\theta_{01}=2{,}5$ $k_a=1$	$\theta_{02}=0{,}7$ $k_l=1$	$Sk_1=1$ $Sk_2=0{,}2$ $k_\lambda=0{,}2$	1,801	1,803	0,11

into the relation

$$W=\frac{\pi^2}{4}\cdot\frac{\sqrt{k_a}\,k_l^2}{4\left(k_\lambda+\sqrt{k_a}\,k_l^2\right)\exp(-\mathrm{Sk}_1)\exp(-\mathrm{Sk}_2)}. \tag{10}$$

Thus, (9) and (10) characterize approximately the regular mode during a transient radiative interaction between bodies.

On the basis of these relations, it is possible to develop an engineering method of calculating the temperature fields after the initial stage of heat transfer. We rewrite Eq. (9) as

$$E(\mathrm{Fo})=E_0\exp(-C+W\,\mathrm{Fo}), \tag{11}$$

where $C = \ln(E_0\mathrm{Sk}_2/W(1-\theta_{02}))$ is a constant found from the condition that the mean-integral temperature [3] approaches the singular equilibrium temperature when $\mathrm{Fo}\to\infty$. The formal solution to system (2)-(6) is well known [2]:

$$\theta_1(X,\,\mathrm{Fo})=\theta_{01}-\frac{\mathrm{Sk}_1}{k_ak_l}\int_0^{\mathrm{Fo}}E(\mathrm{Fo})\,d\,\mathrm{Fo}+\frac{2\mathrm{Sk}_1}{k_ak_l}\sum_{n=1}^{\infty}(-1)^n\cos\frac{\mu_n}{k_l}(k_l+X)\exp\left[-\frac{\mu_n^2\,\mathrm{Fo}}{k_ak_l^2}\right]$$
$$\times\int_0^{\mathrm{Fo}}E(\mathrm{Fo})\exp\left[\frac{\mu_n^2\,\mathrm{Fo}}{k_ak_l^2}\right]d\,\mathrm{Fo};$$

$$\theta_2(X,\,\mathrm{Fo})=\theta_{02}+\mathrm{Sk}_2\int_0^{\mathrm{Fo}}E(\mathrm{Fo})\,d\,\mathrm{Fo}+2\mathrm{Sk}_2\sum_{n=1}^{\infty}(-1)^n\cos\mu_n(1-X)\exp\left[-\mu_n^2\,\mathrm{Fo}\right]\int_0^{\mathrm{Fo}}E(\mathrm{Fo})\exp\left[\mu_n^2\,\mathrm{Fo}\right]d\,\mathrm{Fo},$$

where $\mu_n = \pi n$.

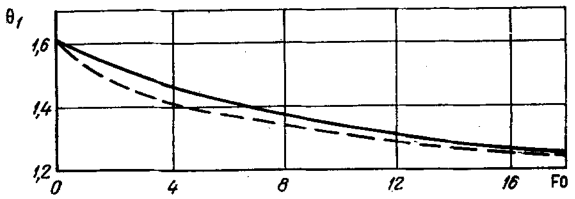

Fig. 3. Comparison between test values of the temperature (solid line) and analytical values of the temperature (dashed line), for $\theta_{01} = 1.62$, $\theta_{02} = 0.762$, $k_a = 0.57$, $k_\lambda = 1.656$, $k_l = 0.4021$, $\mathrm{Sk}_2 = 0.0133$.

Inserting the values of E(Fo) in accordance with (11) yields

$$\theta_1(X,\ \mathrm{Fo}) = \theta_{01} + \frac{E_0\,\mathrm{Sk}_1 \exp(-C)}{W k_a k_l}\left[\exp(-W\,\mathrm{Fo}) - 1\right] - 2E_0 k_l\,\mathrm{Sk}_1 \exp(-C) \sum_{n=1}^{\infty} (-1)^n \frac{\cos\left[\frac{\mu_n}{k_l}(k_l + X)\right]}{\mu_n^2 - W k_a k_l^2}$$

$$\times \left[\exp(-W\,\mathrm{Fo}) - \exp\left(-\frac{\mu_n^2}{k_a k_l^2}\,\mathrm{Fo}\right)\right], \qquad (12)$$

$$\theta_2(X,\ \mathrm{Fo}) = \theta_{02} - \frac{E_0\,\mathrm{Sk}_2 \exp(-C)}{W}\left[\exp(-W\,\mathrm{Fo}) - 1\right]$$

$$+ 2E_0\,\mathrm{Sk}_2 \exp(-C) \sum_{n=1}^{\infty} (-1)^n \frac{\cos\left[\mu_n (1 - X)\right]}{\mu_n^2 - W}\left[\exp(-W\,\mathrm{Fo}) - \exp(-\mu_n^2\,\mathrm{Fo})\right]. \qquad (13)$$

The temperature field based on Eq. (12) and based on measurements is shown in Fig. 3.

LITERATURE CITED

$\theta = T/T_E$	is the dimensionless temperature;
T_E	is the equilibrium temperature;
$k_l = l/R$	is the relative thickness of first layer;
$k_a = a_2/a_1$	is the ratio of thermal diffusivities;
$k_\lambda = \lambda_1/\lambda_2$	is the ratio of thermal conductivities;
$\mathrm{Sk}_i = \sigma T_E^3 R/\lambda_i$	is the Stark number;
σ	is the referred emissivity;
$\mathrm{Fo} = a_2\tau/R^2$	is the Fourier number (dimensionless time)

Subscripts

1 and 2 denote the first and second body respectively.

LITERATURE CITED

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